1.3 NOTES Solving Absolute Value Equations & Inequalities
Systems of Absolute Value Equations

***Solving Absolute Value Equations**

Absolute Value Equations usually have 2 answers. This is because to get rid of the absolute value bars we have to rewrite the equation as two separate linear equations.

Ex: $|x - 3| = 27$ Rewrite the equation as 2 different equations.

$x - 3 = 27$ and $x - 3 = -27$ *Think about which numbers have an absolute value of 27.*

* always = positive

The steps to solve an absolute value equation are:

1. **Isolate the absolute value first**
2. **Rewrite the equation as two separate linear equations**
3. **Solve each equation individually to get the two answers**
4. **Make sure to check your answers!**

Ex 1) Solve $|x + 2| = 7$

$x + 2 = 7$ and $x + 2 = -7$

$-2 \quad -2 \qquad -2 \quad -2$

$x = 5$ $\qquad$ $x = -9$

$|5+2| = 7$ ✓ $\qquad$ $|-9+2| = 7$

$|-7| = 7$ ✓

Ex 2) Solve $|x + 8| - 5 = 2$

$+5 \quad +5$

$|x+8| = 7$

$x + 8 = 7$ $\qquad$ $x + 8 = -7$

$-8 \quad -8 \qquad -8 \quad -8$

$x = -1$ $\qquad$ $x = -15$

$|-1+8| - 5 = 2$ ✓ $\qquad$ $|-15+8| - 5 = 2$ ✓

Ex 3) Solve $3|-8x| + 8 = 80$

$-8 \quad -8$

$\frac{3|-8x|}{3} = \frac{72}{3}$

$|-8x| = 24$

$-8x = 24$ $\qquad$ $-8x = -24$

$\frac{-8x}{-8} = \frac{24}{-8} \qquad \frac{-8x}{-8} = \frac{-24}{-8}$

$x = -3$ ✓ $\qquad$ $x = 3$ ✓

Ex 4) Solve $\left[\frac{|7x+4|}{8} = 3\right] 8$

$|7x+4| = 24$

$7x + 4 = 24$ $\qquad$ $7x + 4 = -24$

$-4 \quad -4 \qquad -4 \quad -4$

$\frac{7x}{7} = \frac{20}{7} \qquad \frac{7x}{7} = \frac{-28}{7}$

$x = \frac{20}{7}$ ✓ $\qquad$ $x = -4$ ✓

YOU TRY: Solve the following absolute value problems

1. $|-2x + 6| = 6$

2. $-5|3 + 4x| = -115$

3. $-5|5x - 5| + 2 = -73$

$-2 \quad -2$

$\frac{-5|5x-5|}{-5} = \frac{-75}{-5}$

$|5x-5| = 15$

$5x - 5 = 15$ $\qquad$ $5x - 5 = -15$

$+5 \quad +5 \qquad +5 \quad +5$

$5x = 20$ $\qquad$ $5x = -10$ ✓

$x = 4$ ✓ $\qquad$ $x = -2$

***Solving Absolute Value Inequalities**

Ex: If $|x| \leq 3$ that means that its distance from zero is less than 3 spaces.
What number(s) is exactly three spaces from zero?

What are other numbers that are less than 3 spaces away from zero?

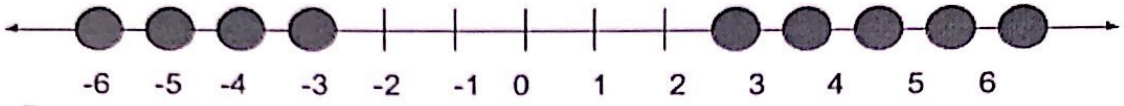
Plot this on a number line:

Where do all these numbers seem to lay? inside

Ex: If $|x| \geq 3$ that means that its distance from zero is more than 3 spaces.
What number(s) is exactly three spaces from zero?

What are other numbers that are more than 3 spaces away from zero?

Plot this on a number line:



Where do these points seem to lie? outside

We call these two situations compound inequalities. These two types are called ***and and ***or*** statements.

And: This is an in-between situation. Your answer would be written **-# < x < #**

Or: This is the "going out" situation. Your answer would be written **x < # or x > #**

All absolute value inequalities make an **and** or an **or** statement. We know which by what the sign is.

Less Thand – Less than Absolute values make and statements

Greator – Greater than Absolute values make or statements.

* flip inequality for negative

ALWAYS ISOLATE THE ABSOLUTE VALUE BEFORE YOU BREAK IT UP AND SOLVE!!

Ex 1) Solve $|x - 2| \leq 7$

$x-2 \leq 7$ & $x-2 \geq -7$

$x \leq 9$ and $x \geq -5$

Ex 2) Solve $4|x - 3| - 5 > 3$

$|x-3| > 2$

$x-3 > 2$ or $x-3 < -2$

$x > 5$ or $x < 1$

Ex 3) Solve $5|-6x| - 3 < 80$

$|-6x| < 16.6$

$-6x < 16.6$ & $-6x > -16.6$

$x > -2.77$ and $x < 2.77$

Ex 4) Solve $\frac{|7x-4|}{-8} < 3$

$|7x-4| > -24$

~~$7x-4 < -24$~~ ~~$7x-4 > 24$~~

✓ ~~$x < -2.86$~~ and ~~$x > 4$~~

YOU TRY: Solve the following absolute value problems

1. $|6x+2|<9$

$6x+2<9$ ⅋ $6x+2>-9$

$-2 \quad -2$

$6x<7$ $\qquad 6x>-11$

$x<\frac{7}{6}$ ⅋ $x>\frac{-11}{6}$

2. $-5|3+4x| \geq -115$

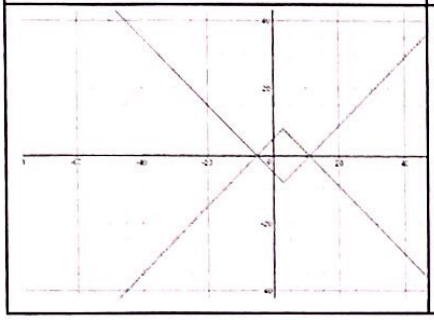
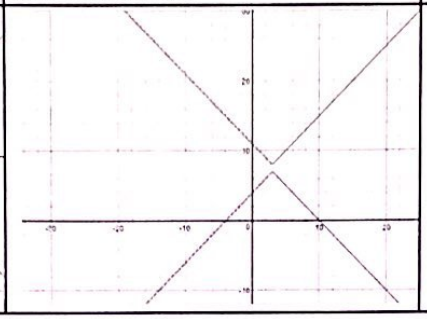
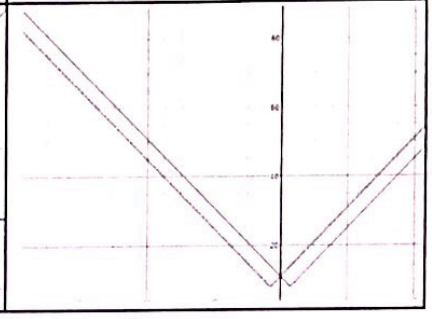
$|3+4x| \leq 23$

$3+4x \leq 23$ ⅋ $3+4x \geq -23$

$x \leq 5$ ⅋ $x \geq -6.5$

***Systems of Absolute Value Equations**

Graph on the calculator!

Two Solutions	No Solutions	One Solution
		

EX4. $y=|x-3|-4$

$y=-|x-3|+6$

math → num → abs(

(8, 1)

(-2, 1)