NOTES: Inverses of Functions

Inverses are a very important mathematical concept! We use them in almost every problem because they are what "undo" operations. When we solve for a variable like x in $10 = 2x$, we are using inverses.

1. Claim: $y = 3x$ and $y = \frac{x}{3}$ are inverse functions. Explain why this is true.

multiplication & division undo each other

2. Fill in the tables for $y = 3x$ and $y = \frac{x}{3}$.

$y = 3x$

x	y
−2	-6
−1	-3
0	0
1	3
2	6

x Domain: $(-\infty, \infty)$ ℝ

y Range: $(-\infty, \infty)$

$y = \frac{x}{3}$

x	y
−6	-2
−3	-1
0	0
3	1
6	2

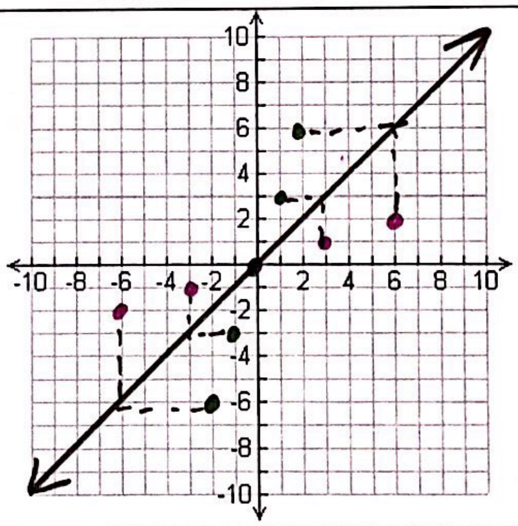
Domain: $(-\infty, \infty)$

Range: $(-\infty, \infty)$

3. What do you notice about the x and y values of these two functions?

x values are y values of inverse & y values are x values of inverse

4. For each function, plot the points from the tables on the graph using a different color.



5. How do the **graphs** of the two functions relate to each other and the line $y = x$?

reflect over $y=x$

Summary

6. Functions and their inverses are reflections across the line $y = x$.

7. We can find coordinates of an inverse function by switching the x and y values.

8. Notation: We denote the inverse of $f(x)$ as $f^{-1}(x)$ or y^{-1}

Apply Your Knowledge: *For each of the following, find the coordinates of the inverse function.*

1. $f(x) = (2,4),\ (-5,3),\ (-2,-3)$

$f^{-1}(x) =$ (4,2), (3,-5), (-3,-2)

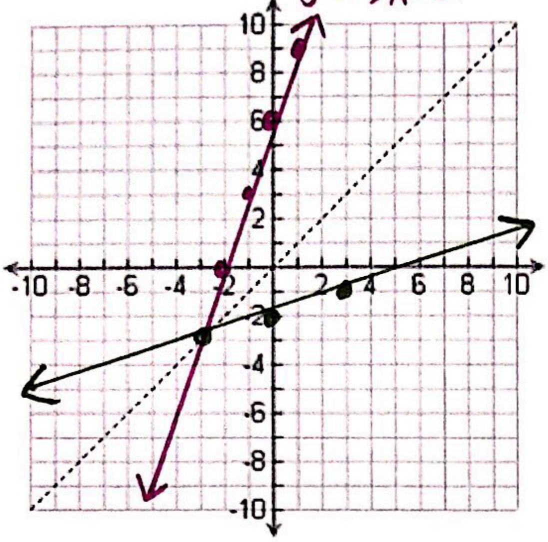
2. $h(x) = (0,0),\ (-2,4),\ (-1,-4),\ (2,2),\ (-3,-3),\ (-4,4)$

$h^{-1}(x) =$ (0,0), (4,-2), (-4,-1), (2,2), (-3,-3), (4,-4)

3. Graph the function $f(x) = 6 + 3x$ and its inverse, $f^{-1}(x) = \frac{x-6}{3}$.

y = 3x+6

$y^{-1} = \frac{1}{3}x - 2$



What do you notice?

reflects over y=x

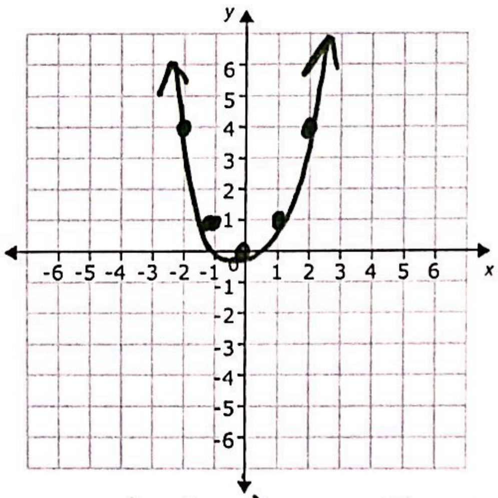
More on Inverses…

1. In previous classes, you have used the "***Vertical Line Test.***" What does this test prove?

If it is a function

Earlier, we found that the graph of the inverse of a function can be found by reflecting over the line y = x (switching the x and y coordinates). When we do this, we may not always end up with a "function."

2. Sketch the graph of $h(x) = x^2$.

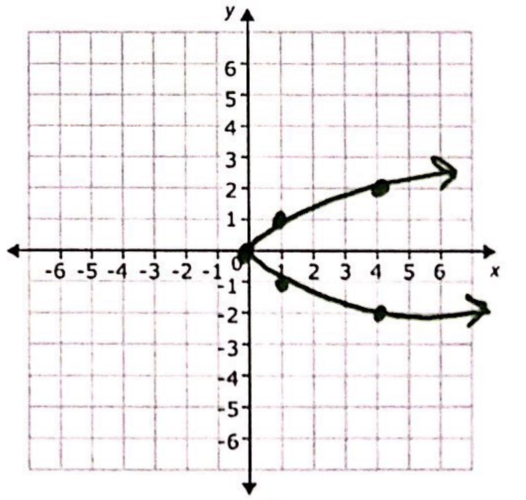


Domain: $(-\infty, \infty)$ Range: $[0, \infty)$

3. Is $h(x) = x^2$ a function? How do you know?

yes! passes VLT

4. Sketch the graph of the inverse $h^{-1}(x) = \pm\sqrt{x}$ by switching the x and y coordinates..



Domain: $[0, \infty)$ Range: $(-\infty, \infty)$

5. Is $h^{-1}(x)$ a function? How do you know?

No! Fails VLT