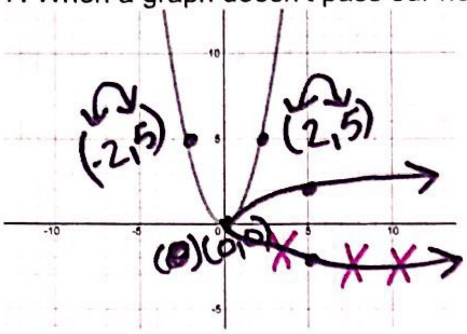
6. **Critical Thinking**: How could we determine if a function has an inverse without sketching its graph? Could we create a different "**test**"?

Horizontal Line Test

VLT ✓
HLT X

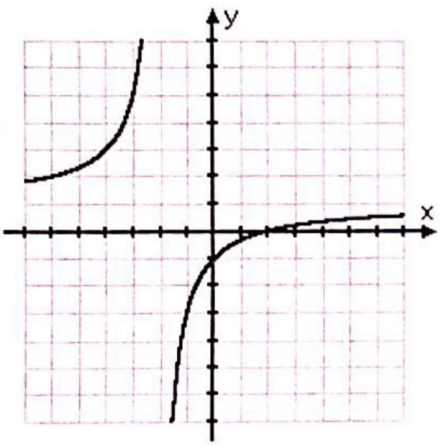
7. When a graph doesn't pass our new "**test**", we can still find the inverse but we have to restrict the domain.



$f^{-1}(x)$

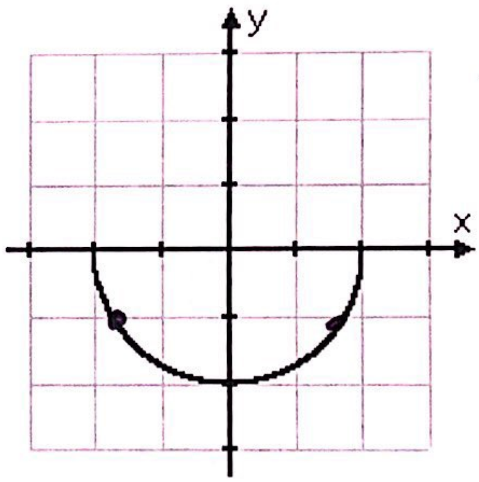
Check your Understanding: *Given the following graphs, determine if its inverse is function. If needed, restrict the domain.*

1.



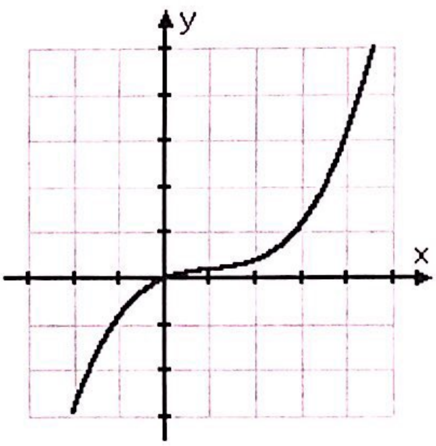
yes!

2.



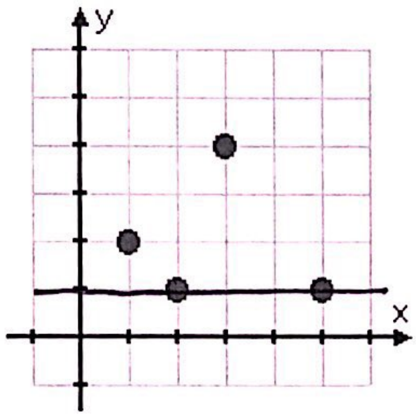
No!

3.



yes!

4.



no!

5. A function has the following points: $(2,3)$, $(5,1)$, $(6,5)$, $(-3,1)$, $(-5,6)$. Is the inverse a function? Why or why not?

no, y's repeat themselves

Steps for Writing an Inverse Function
We can find the expression for the Inverse function using the following steps:

1. Change "$f(x)$" notation to "y" notation

2. Switch the x and the y variables in the function

3. Solve the equation for y.

4. Replace y with $f^{-1}(x)$

Find the inverse of $f(x) = 3x - 2$
$f(x) = 3x - 2$
$y = 3x - 2 \longrightarrow x = 3y - 2$
$x + 2 = 3y$
$\frac{x+2}{3} = y$
$f^{-1}(x) = \frac{x+2}{3}$

Find the inverse of each.

1. $f(x) = -\frac{1}{2}x + 1$

$x = \frac{-1}{2}y + 1$

$-1 \qquad -1$

$(-2)(x-1) = \left(\frac{-1}{2}y\right)(-2)$

$\boxed{y^{-1} = -2x + 2}$

2. $f(x) = x^3 - 2$

$x = y^3 - 2$

$+2 \qquad +2$

$\sqrt[3]{x+2} = \sqrt[3]{y^3}$

$\boxed{y^{-1} = \sqrt[3]{x+2}}$

3. $f(x) = \sqrt[5]{x} - 2$

$y = \sqrt[5]{x} - 2$

$x = \sqrt[5]{y} - 2$

$+2 \qquad +2$

$(x+2)^5 = \sqrt[5]{y}^5$

$\boxed{y^{-1} = (x+2)^5}$

4. $f(x) = 4\sqrt{x-1}$

$x = 4\sqrt{y-1}$

$\left(\frac{x}{4}\right)^2 = \sqrt{y-1}^2$

$\frac{x^2}{16} = y - 1$

$\boxed{y^{-1} = \frac{x^2}{16} + 1}$

1.5: Function Operations

$(f+g)(x) = f(x) + g(x)$

$(f-g)(x) = f(x) - g(x)$

$(f \cdot g)(x) = f(x) \cdot g(x)$

$\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}$

- Ex 1) Given $f(x) = 7x-6$ & $g(x) = 8x^2-3x+4$

a) $(f+g)(x) = f(x) + g(x)$

$= 7x-6+8x^2-3x+4$

$= \boxed{8x^2+4x-2}$

b) $(f-g)(x) = f(x) - g(x)$

$= 7x-6-(8x^2-3x+4)$

$= 7x-6-8x^2+3x-4$

$= \boxed{-8x^2+10x-10}$

c) $(g-f)(x) = g(x) - f(x)$

$= 8x^2-3x+4-(7x-6)$

$= 8x^2-3x+4-7x+6$

$= \boxed{8x^2-10x+10}$

d) $(g \cdot f)(x) = g(x) \cdot f(x)$

$= (8x^2 - 3x + 4)(7x - 6)$

$= 56x^3 - 48x^2 - 21x^2 + 18x + 28x - 24$

$= \boxed{56x^3 - 69x^2 + 46x - 24}$

e) $\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}$

$= \boxed{\frac{7x-6}{8x^2-3x+4}}$

f) $(f-g)(17) = f(17) - g(17)$

$= 113 - 2265$

$= \boxed{-2152}$

$f(17) = 7(17) - 6$

$= 113$

$g(17) = 8(17)^2 - 3(17) + 4$

$= 2265$