

1.6: Compositions of Functions

$$(f \circ g)(x) \rightarrow f(g(x)) \quad \text{"f of g of x"}$$

- putting one function into another
- * NOT multiplication *

Given $f(x) = x^2 + 1$ $g(x) = x - 5$, find:

Ex 1) $(f \circ g)(2) \rightarrow f(g(2))$

$$\begin{aligned} &= f(2 - 5) \\ &= f(-3) \\ &= (-3)^2 + 1 \\ &= \boxed{10} \end{aligned}$$

* inside \rightarrow outside

Ex 2) $(g \circ f)(2) = g(f(2))$

$$\begin{aligned} &= g(2^2 + 1) \\ &= g(5) \\ &= 5 - 5 \\ &= \boxed{0} \end{aligned}$$

$$\begin{aligned}
 \text{Ex 3) } f(g(x)) &= f(x-5) \\
 &= (x-5)^2 + 1 \\
 &= (x-5)(x-5) + 1 \\
 &= x^2 - 5x - 5x + 25 + 1 \\
 &= \boxed{x^2 - 10x + 26}
 \end{aligned}$$

$$\begin{aligned}
 \text{Ex 4) } g(f(x)) &= g(x^2 + 1) \\
 &= (x^2 + 1) - 5 \\
 &= x^2 + 1 - 5 \\
 &= \boxed{x^2 - 4}
 \end{aligned}$$

Given $f(x) = 6x$ & $g(x) = 2x^2 + x - 3$, find:

$$\begin{aligned}
 \text{Ex 5) } (f \circ g)(x) &= f(g(x)) \\
 &= f(2x^2 + x - 3) \\
 &= 6(2x^2 + x - 3) \\
 &= \boxed{12x^2 + 6x - 18}
 \end{aligned}$$

$$\begin{aligned}
 \text{Ex 6) } (g \circ f)(x) &= g(f(x)) \\
 &= g(6x) \\
 &= 2(6x)^2 + 6x - 3 \\
 &= 2(36x^2) + 6x - 3 \\
 &= \boxed{72x^2 + 6x - 3}
 \end{aligned}$$

Proving two functions are inverses

Ex 7) Prove $f(x) = 2x + 3$ \mapsto $g(x) = \frac{x-3}{2}$ are inverses

$$\begin{aligned} f(g(x)) &= f\left(\frac{x-3}{2}\right) \\ &= 2\left(\frac{x-3}{2}\right) + 3 \\ &= x - \cancel{3} + \cancel{3} \\ &= x \quad \checkmark \end{aligned}$$

$$\begin{aligned} g(f(x)) &= g(2x+3) \\ &= \frac{(2x+\cancel{3})-\cancel{3}}{2} \\ &= \frac{2x}{2} \\ &= x \quad \checkmark \end{aligned}$$

Ex 8) Prove that $a(x) = 6\sqrt{x} - 2$ \mapsto $b(x) = \frac{(x+2)^2}{36}$ are inverses

$$\begin{aligned} a(b(x)) &= a\left(\frac{(x+2)^2}{36}\right) \\ &= 6\sqrt{\frac{(x+2)^2}{36}} - 2 \\ &= 6 \cdot \frac{\sqrt{(x+2)^2}}{\sqrt{36}} - 2 \\ &= \cancel{6} \cdot \frac{x+2}{\cancel{6}} - 2 \\ &= x + \cancel{2} - \cancel{2} \\ &= x \quad \checkmark \end{aligned}$$

$$\begin{aligned} b(a(x)) &= b(6\sqrt{x} - 2) \\ &= \frac{((6\sqrt{x} - 2) + 2)^2}{36} \\ &= \frac{(6\sqrt{x})^2}{36} \\ &= \frac{\cancel{36}x}{\cancel{36}} \\ &= x \quad \checkmark \end{aligned}$$