1.6: Compositions of Functions

$(f \circ g)(x) \rightarrow f(g(x))$ "f of g of x"

- putting one function into another

* NOT multiplication *

Given $f(x) = x^2 + 1$ & $g(x) = x - 5$, find:

* inside → outside

Ex 1) $(f \circ g)(2) \rightarrow f(g(2))$

$= f(2-5)$

$= f(-3)$

$= (-3)^2 + 1$

$= \boxed{10}$

Ex 2) $(g \circ f)(2) = g(f(2))$

$= g((2)^2 + 1)$

$= g(5)$

$= 5 - 5$

$= \boxed{0}$

Ex 3) $f(g(x)) = f(x-5)$

$= (x-5)^2 + 1$

$= (x-5)(x-5) + 1$

$= x^2 - 5x - 5x + 25 + 1$

$= \boxed{x^2 - 10x + 26}$

Ex 4) $g(f(x)) = g(x^2+1)$

$= (x^2+1) - 5$

$= x^2 + 1 - 5$

$= \boxed{x^2 - 4}$

Given $f(x) = 6x$ & $g(x) = 2x^2 + x - 3$, find:

Ex 5) $(f \circ g)(x) = f(g(x))$

$= f(2x^2 + x - 3)$

$= 6(2x^2 + x - 3)$

$= \boxed{12x^2 + 6x - 18}$

Ex 6) $(g \circ f)(x) = g(f(x))$

$= g(6x)$

$= 2(6x)^2 + 6x - 3$

$= 2(36x^2) + 6x - 3$

$= \boxed{72x^2 + 6x - 3}$

Proving two functions are inverses

Ex 7) Prove $f(x)=2x+3$ & $g(x)=\frac{x-3}{2}$ are inverses

$f(g(x)) = f\left(\frac{x-3}{2}\right)$

$= 2\left(\frac{x-3}{2}\right)+3$

$= x-3+3$

$= x$ ✓

$g(f(x)) = g(2x+3)$

$= \frac{(2x+3)-3}{2}$

$= \frac{2x}{2}$

$= x$ ✓

Ex 8) Prove that $a(x)=6\sqrt{x}-2$ & $b(x)=\frac{(x+2)^2}{36}$ are inverses

$a(b(x)) = a\left(\frac{(x+2)^2}{36}\right)$

$= 6\sqrt{\frac{(x+2)^2}{36}}-2$

$= 6\cdot\frac{\sqrt{(x+2)^2}}{\sqrt{36}}-2$

$= 6\cdot\frac{x+2}{6}-2$

$= x+2-2$

$= x$ ✓

$b(a(x)) = b(6\sqrt{x}-2)$

$= \frac{((6\sqrt{x}-2)+2)^2}{36}$

$= \frac{(6\sqrt{x})^2}{36}$

$= \frac{36x}{36}$

$= x$ ✓