2.2: Synthetic Division

Long division → works every time

synthetic division → only works when the leading coefficient of divisor is 1. $(x \pm a)$

Ex 1) $(x^3 - 4x^2 + 6x - 4) \div (x-2)$ ← factor of polynomial!

Step 1) Set divisor = 0 & solve

$$x - 2 = 0$$
$$+2 \quad +2$$
$$x = 2$$

Step 2) put solution in a box. Line up coefficients of dividend next to box

$$\begin{array}{c|cccc}
2 & 1 & -4 & 6 & -4 \\
 & \downarrow & +2 & -4 & 4 \\
\hline
x & 1 & -2 & 2 & \boxed{0}
\end{array}$$

← remainder

Step 3) add columns

Step 4) multiply w/ box & put # in next column

Step 5) repeat

Step 6) write solution w/ x down by 1 exponent

$$\boxed{x^2 - 2x + 2}$$

Ex 2) $(5x^4 + 3x^3 - 2x^2 + 7) \div (x+6)$

$+0x$

$x+6=0$
$x=-6$

$$\begin{array}{r|rrrrr} -6 & 5 & 3 & -2 & 0 & 7 \\ & \downarrow & -30 & 162 & -960 & 5760 \\ \hline & 5 & -27 & 160 & -960 & 5767 \end{array}$$

$$\boxed{5x^3 - 27x^2 + 160x - 960 + \frac{5767}{x+6}}$$

Ex 3) $(-9m^2 + 18m + 9m^5 - 18m^4 + 8) \div (m-2)$

$+0m^3$

$(9m^5 - 18m^4 + 0m^3 - 9m^2 + 18m + 8) \div (m-2)$

$m-2=0$
$m=2$

$$\begin{array}{r|rrrrrr} 2 & 9 & -18 & 0 & -9 & 18 & 8 \\ & \downarrow & 18 & 0 & 0 & -18 & 0 \\ \hline & 9 & 0 & 0 & -9 & 0 & 8 \end{array}$$

$9m^4 + \cancel{0m^3} + \cancel{0m^2} - 9m + \cancel{0} + \frac{8}{m-2}$

$$\boxed{9m^4 - 9m + \frac{8}{m-2}}$$

Remainder Theorem: When a polynomial $P(x)$ is divided by $x-a$, the remainder is $P(a)$.

Factor Theorem: For a polynomial $P(x)$, $x-a$ is a factor if & only if $P(a)=0$.

Ex 4) a) Is $x-4$ a factor of $x^{10} + 4x^9 - 21x^8$?

$x-4=0$
$x=4$

$P(4) = (4)^{10} + 4(4)^9 - 21(4)^8 = 720896$

$\boxed{x-4 \text{ is not a factor!}}$

b) is $x+7$ a factor? Remainder $= 0$

$\boxed{x+7 \text{ is a factor!}}$

Ex 5) find all solutions of $x^3-x^2-10x-8$ given $x=-1$ is a solution.

$$\begin{array}{r|rrrr} -1 & 1 & -1 & -10 & -8 \\ & \downarrow & -1 & 2 & 8 \\ \hline & 1 & -2 & -8 & \boxed{0} \end{array}$$

x^2-2x-8

factor!

$(x-4)(x+2)$

$x-4=0$ $\boxed{x=4}$

$x+2=0$ $\boxed{x=-2}$

$\boxed{x=-1}$