

Unit 2 - Polynomials

what is a polynomial?

Extrema

Increasing / Decreasing

Positive / Negative

Zeros

Polynomial Vocabulary

Multiplicity

End Behavior

polynomial - an expression in the form

$$a_n x^n + a_{n-1} x^{n-1} + \dots + a_2 x^2 + a_1 x + a_0$$

\uparrow
coefficient

$n \in \text{positive, integer}$

Are polynomials

$$\frac{1}{2} x^2$$

$$-4x^3 + 7x^2 - 6x + 1$$

$$12$$

Are not polynomials

$$x^{-1} \text{ can't have negative exponents}$$

$$5x^{\frac{1}{2}} - 8 \text{ can't have fractional exponents}$$

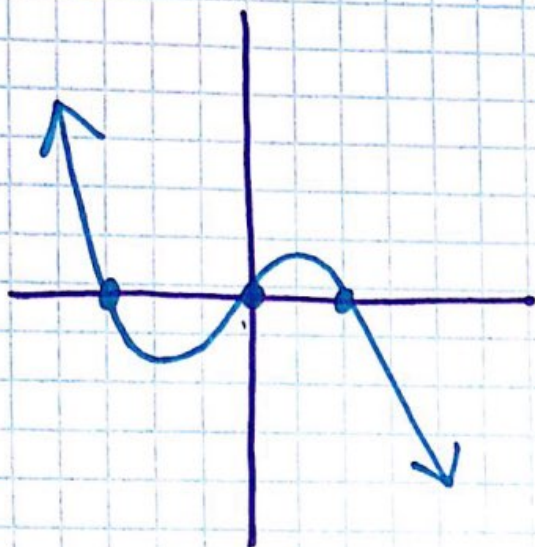
$$11^x \text{ can't have variable exponents}$$

what is a polynomial?

Zero - a number "k" is a zero of a polynomial if $f(k) = 0$

zeros are also called roots or solutions

zeros are the x-intercepts of the graph.



zeros: $x = -3$
 $x = 0$
 $x = 2$

factored form:
 $f(x) = (x+3)(x)(x-2)$

Zeros



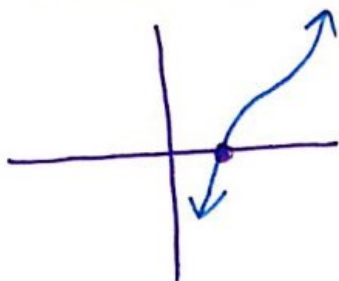
zeros: $x = -3$
 $x = -2$
 $x = -1$
 $x = 2$

factored form:
 $f(x) = (x+3)(x+2)(x+1)(x-2)$

Multiplicity: how often a zero occurs

multiplicity of 1
(x+a)

- cross x-axis
- "straight thru"

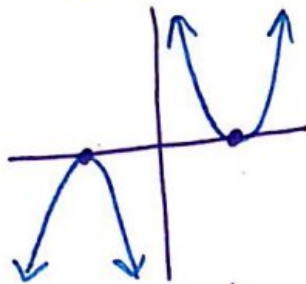


single root

multiplicity of 2
(or other even #)

$$(x+a)^2$$

- bounce on x-axis
- "bounce"

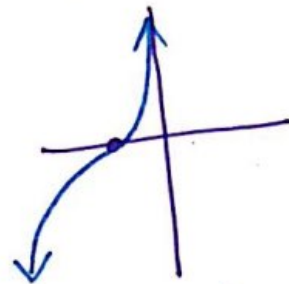


double root

multiplicity of 3
(or other odd #)

$$(x+a)^3$$

- flatten as it crosses x-axis
- "flatten"



triple root

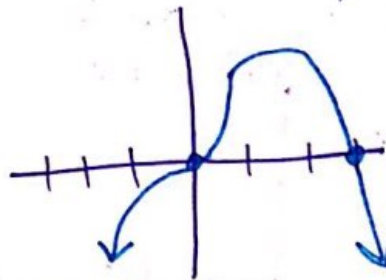
$$f(x) = (x-13)^2(x+3)^8(x-4)$$

- zeros: $x=13$ mult: 2
 $x=-3$ mult: 8
 $x=4$ mult: 1



- zeros: $x=-2$ mult: 2
 $x=1$ mult: 1

factored form: $(x+2)^2(x-1) = y$



- zeros: $x=0$ mult: 3
 $x=3$ mult: 1

factored form: $y = x^3(x-3)$

Multiplicity

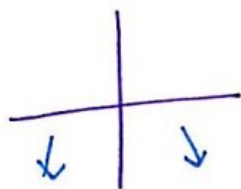
End Behavior: what the graph does at each end

Look at leading term of Polynomial

Leading Coefficient

| | | Positive | negative |
|----------------------------|------|---|--|
| D e g r e e | even | <p>as $x \rightarrow -\infty$ $f(x) \rightarrow \infty$</p> <p>as $x \rightarrow \infty$ $f(x) \rightarrow \infty$</p> <p>EX: $2x^2 = y$</p> | <p>as $x \rightarrow -\infty$ $f(x) \rightarrow -\infty$</p> <p>as $x \rightarrow \infty$ $f(x) \rightarrow -\infty$</p> <p>EX: $-3x^2 = y$</p> |
| | odd | <p>as $x \rightarrow -\infty$ $f(x) \rightarrow -\infty$</p> <p>as $x \rightarrow \infty$ $y \rightarrow \infty$ $f(x) \rightarrow \infty$</p> <p>EX: $x = y$</p> | <p>as $x \rightarrow -\infty$ $f(x) \rightarrow \infty$</p> <p>as $x \rightarrow \infty$ $f(x) \rightarrow -\infty$</p> <p>EX: $y = -x$</p> |

$f(x) = -7x^4 - 6x + 2$



as $x \rightarrow -\infty$
 $f(x) \rightarrow -\infty$

as $x \rightarrow \infty$
 $f(x) \rightarrow -\infty$

End Behavior