GUIDED NOTES: Graphs of Polynomials

EX1. $f(x) = 5x^3 - x^2 - 5x + 1$

Factored Equation: $f(x) = (x-1)(x-\frac{1}{5})(x+1)$

Zeroes (with multiplicity):

$x = 1$ m1

$x = \frac{1}{5}$ m1

$x = -1$ m1

Intervals for Positive/Negative:

pos: $(-1, \frac{1}{5})$ & $(1, \infty)$

neg: $(-\infty, -1)$ & $(\frac{1}{5}, 1)$

Extrema:

$(-0.39, 2.49)$ relative max

$(0.55, -1.21)$ relative min

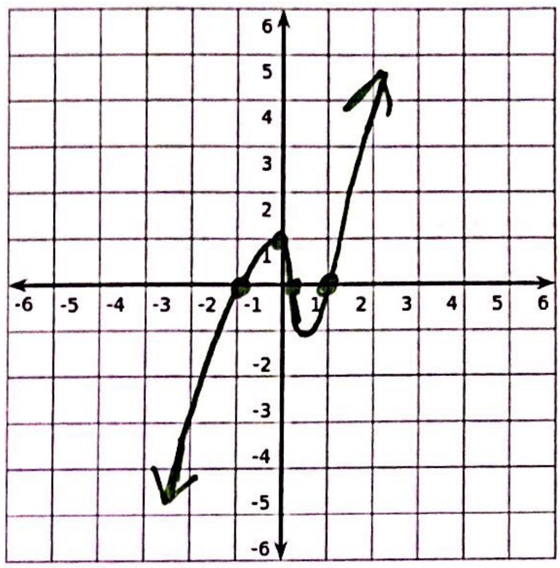
Intervals for Increasing/Decreasing:

inc: $(-\infty, -0.39)$ & $(0.55, \infty)$

dec: $(-0.39, 0.55)$

End Behavior: as $x \to -\infty$, $f(x) \to -\infty$

as $x \to \infty$, $f(x) \to \infty$



Y-intercept: $(0, 1)$

EX2. $f(x) = 3x^4 - 6x^2 + 3$

Factored Equation:

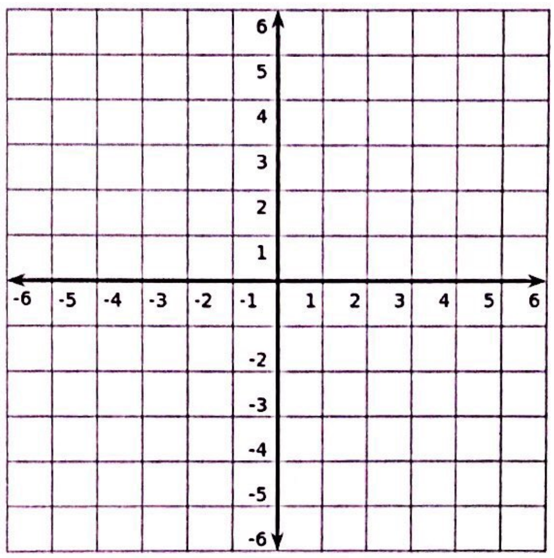
Zeroes (with multiplicity):

Intervals for Positive/Negative:

Extrema:

Intervals for Increasing/Decreasing:

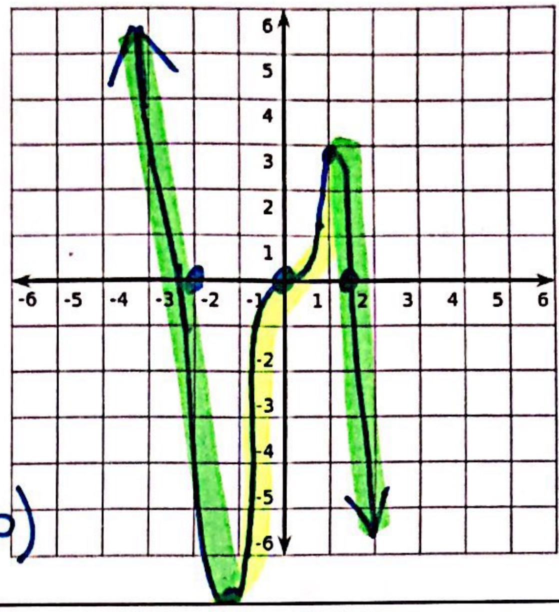
End Behavior:



Y-intercept:

(don't do all together!!!)

EX3. $f(x) = -2x^5 - x^4 + 6x^3$



Factored Equation:
$f(x) = x^3(x+2)(-2x+3)$

Zeroes (with multiplicity):
$x=0$ m 3
$x=-2$ m1
$x=3/2$ m1

Intervals for Positive/Negative:
pos: $(-\infty,-2)$ & $(0, 3/2)$
neg: $(-2,0)$ & $(\frac{3}{2},\infty)$

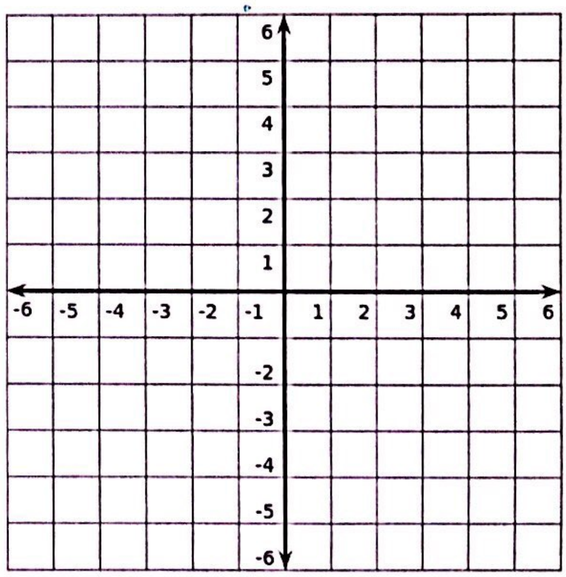
Extrema:
$(-1.49,-10.09)$ rel. min
$(1.06, 3.22)$ rel max

Intervals for Increasing/Decreasing:
inc: $(-1.49, 1.06)$
dec: $(-\infty,-1.49)$ & $(1.06,\infty)$

End Behavior:
as $x\to-\infty$, $f(x)\to\infty$
as $x\to\infty$, $f(x)\to-\infty$

Y-intercept:
$(0,0)$

EX4. $f(x) = -2x^4 + x^3 + 6x^2$



Factored Equation:

Zeroes (with multiplicity):

Intervals for Positive/Negative:

Extrema:

Intervals for Increasing/Decreasing:

End Behavior:

Y-intercept:

$(-2x^5 - x^4 + 6x^3) \div (x+2)$

$+0x^2 + 0x + 0$

$x+2=0$

$x=-2$

$$\begin{array}{r|rrrrrr} -2 & -2 & -1 & 6 & 0 & 0 & 0 \\ & \downarrow & 4 & -6 & 0 & 0 & 0 \\ \hline & -2 & 3 & 0 & 0 & 0 & \cancel{0} \end{array} \rightarrow (x+2)(-2x^4+3x^3$$

$$\begin{array}{r|rrrrr} 0 & -2 & 3 & 0 & 0 & 0 \\ & \downarrow & 0 & 0 & 0 & 0 \\ \hline & -2 & 3 & 0 & 0 & \cancel{0} \end{array} \quad (x+2)(x)(-2x^3+3x^2$$

$$\begin{array}{r|rrrr} 0 & -2 & 3 & 0 & 0 \\ & \downarrow & 0 & 0 & 0 \\ \hline & -2 & 3 & 0 & \cancel{0} \end{array} \rightarrow (x+2)(x^2)(-2x^2+3x$$

$$\begin{array}{r|rrr} 0 & -2 & 3 & 0 \\ & \downarrow & 0 & 0 \\ \hline & -2 & 3 & 0 \end{array} \rightarrow (x+2)(x^3)(-2x+3)$$

RULES FOR TRANSFORMATIONS OF FUNCTIONS If $f(x)$ is the original function, $a > 0$ and $c > 0$:	
Function	**Transformation of the graph of $f(x)$**
$f(x)+c$	Shift $f(x)$ upward c units
$f(x)-c$	Shift $f(x)$ downward c units
$f(x+c)$	Shift $f(x)$ to the left c units
$f(x-c)$	Shift $f(x)$ to the right c units
$-f(x)$	Reflect $f(x)$ in the x-axis
$f(-x)$	Reflect $f(x)$ in the y-axis
$a \cdot f(x),\ a > 1$	Stretch $f(x)$ vertically by a factor of a.
$a \cdot f(x),\ 0 < a < 1$	Shrink $f(x)$ vertically by a factor of a.
$f(ax),\ a > 1$	Shrink $f(x)$ horizontally by a factor of $\frac{1}{a}$.
$f(ax),\ 0 < a < 1$	Stretch $f(x)$ horizontally by a factor of $\frac{1}{a}$.

Practice:

1. Write an equation that will move the graph of the function $y=x^3$ right 4 units and reflect over the x-axis.

$y = -(x-4)^3$

2. The equation $y = (x+3)^2 - 2$ moves the parent function $y = x^2$ right 3 units and down 2 units. **True or False**

left

3. The equation $y = (x-8)^2 + 5$ moves the parent function $y = x^2$ right 8 units and down 5 units. **True or False**

up

4. Write an equation that will move the graph of the function $y=x^4$ left 2 units and up 6 units with a reflection across the x-axis.

$y = -(x+2)^4 + 6$

5. Which equation will shift the graph of $y = x^2$ left 5 units and up 6 units?

~~a.~~ $y = (x+6)^2 - 5$　　b. $y = (x+5)^2 - 6$　　c. $y = (x+5)^2 + 6$　　~~d.~~ $y = (x-5)^2 + 6$

6. Which equation will shift the graph of $y = x^2$ right 8 units and down 4 units?

~~a.~~ $y = (x+8)^2 - 4$　　~~b.~~ $y = (x+4)^2 - 8$　　~~c.~~ $y = (x-4)^2 + 8$　　d. $y = (x-8)^2 - 4$