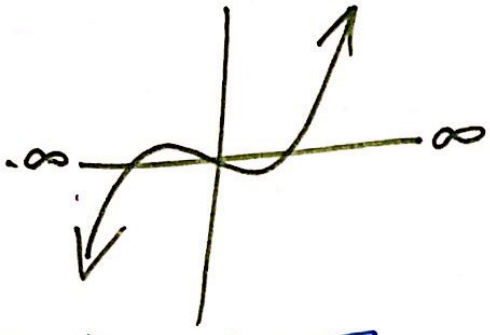
2.6: Solving Polynomial Equations

- End Behavior ($\downarrow,\uparrow$)

as $x \to -\infty$ (left) $y \to -\infty$ (down)

as $x \to \infty$ (right) $y \to \infty$ (up)



- Fundamental Theorem of Algebra: The degree of a polynomial is the number of solutions (real or complex) the equation has

- Ex 1) solve $x^3+3x^2-x+12=0$

1) graph to find x's $x=-4$ m l

2) divide by the factor & solve for other factors

$$\begin{array}{r|rrrr} -4 & 1 & 3 & -1 & 12 \\ & \downarrow & -4 & 4 & -12 \\ \hline & 1 & -1 & 3 & 0 \checkmark \end{array} \rightarrow (\underbrace{1}_{a}x^2 \underbrace{-1}_{b}x \underbrace{+3}_{c})(x+4)$$

$x=-4$ m l, $x=\frac{1\pm i\sqrt{11}}{2}$ m l

quadratic formula $\rightarrow$ $x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$

$x=\frac{-(-1)\pm\sqrt{(-1)^2-4(1)(3)}}{2(1)}$

$x=\frac{1\pm\sqrt{-11}}{2}=x=\frac{1\pm i\sqrt{11}}{2}$

zeroes: $x=-4$, $x=\frac{1+i\sqrt{11}}{2}$, $x=\frac{1-i\sqrt{11}}{2}$

Ex 2) Solve: $x^4+4x^3-4x^2-36x-45=0$

$x=-3$ ml $x=3$ ml

-3 | 1 4 −4 −36 −45

↓ −3 −3 21 45

3 | 1 1 −7 −15 | ✗ → $(x+3)(x^3+1x^2-7x-15)$

↓ 3 12 15

1 4 5 | 0 → $(x+3)(x-3)(x^2+4x+5)$

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$x=\frac{-4\pm\sqrt{4^2-4(1)(5)}}{2(1)}$$

$$x=\frac{-4\pm\sqrt{-4}}{2}$$

$$x=\frac{-4\pm 2i}{2}=-2\pm i$$

Zeroes: $x=3$ ml
$x=3$ ml
$x=-2+i$ ml
$x=-2-i$ ml

you try: solve $x^4 - 6x^3 + 25x^2 - 96x + 144 = 0$

3	1	-6	25	-96	144
	↓	3	-9	48	-144
3	1	-3	16	-48	0
	↓	3	0	48	
	1	0	16	0	

$x^2 + 16 = 0$

$-16 \quad -16$

$\pm\sqrt{x^2} = \pm\sqrt{-16}$

$x = \pm 4i$

zeroes: $x = 3$ m2

$x = 4i$ m1

$x = -4i$ m1

(↑, ↑) as $x \to -\infty$, $y \to \infty$

as $x \to \infty$, $y \to \infty$

Ex 5) solve: $x^7 - 12x^6 + 26x^5 + 224x^4 - 1299x^3 + 2020x^2 + 624x - 2880 = 0$

zeroes: $x = -5$ m1

$x = -1$ m1

$x = 3$ m2

$x = 4$ m3