

4.7 GUIDED NOTES: Graphing Rational Functions Notes

DOMAIN: The domain of a function is the set of all possible X-values

Restrictions are values not included in the domain!

RANGE: The range of a function is the set of all possible y-values

Vertical Asymptotes and Holes:

To find vertical asymptotes: set factor in denominator = 0 & solve.

To find holes: find matching factors in numerator & denominator.

EX1. $y = \frac{x^2-1}{x^2-2x-3} = \frac{(x+1)(x-1)}{(x-3)(x+1)}$
 Set = 0 & solve.

hole: $x+1=0$
 $x = -1$

VA: $x-3=0$
 $x = 3$

x-int: $x-1=0$
 $x = 1$ $(1, 0)$

Horizontal Asymptotes:

To find a horizontal asymptotes, we focus on the degree of the numerator and the denominator!

- If deg denom is greater than deg num, HA is $y=0$.
- If deg denom is less than deg num, there is no HA
- If deg denom is same as deg num, HA is a ratio of leading coefficients

EX3. $y = \frac{(x-3)(x+4)^2}{(x-5)^3(x+2)^2} = \frac{3}{5}$

HA: $y = 0$
 $y = \frac{0x^5}{1x^5}$

EX4. $y = \frac{4x^6+3x^4-4x^2}{3x} = \frac{6}{1}$

HA: none
 $y = \frac{4x^6}{0x^6}$

EX5. $y = \frac{6x-5}{3x+9} = \frac{1}{1}$

HA: $y = \frac{6x}{3x}$
 $y = 2$

For each rational function, determine the

- 1) vertical asymptotes and holes
- 2) domain
- 3) horizontal asymptote
- 4) x-intercepts (Set the top = 0 and solve! Be careful for holes)
- 5) y-intercept (Substitute in 0 for x and solve for y)
- 6) graph!!

$$\text{EX6: } h(x) = \frac{x-2}{x^2-4} = \frac{\cancel{x-2}}{(x+2)\cancel{(x-2)}} = \frac{1}{x+2}$$

note: $x-2=0$
 $x=2 \rightarrow \frac{1}{2+2} = \frac{1}{4}$

$$\boxed{(2, \frac{1}{4})}$$

VA: $x+2=0$
 $x=-2$

Domain: $x \neq \pm 2$

HA: $y=0$

X-int: $x-2=0$

~~x=2~~ none

y-int: $\frac{1}{0+2} = \frac{1}{2}$

$$\boxed{(0, \frac{1}{2})}$$

$$\text{EX7: } f(x) = \frac{x^2+x-6}{-4x^2-16x-12} = \frac{(x+3)(x-2)}{-4(x^2+4x+3)} = \frac{(x+3)(x-2)}{-4(x+3)(x+1)}$$

note: $x+3=0$
 $x=-3$
 $\frac{-3-2}{-4(-3+1)} = \frac{-5}{8}$

$$\boxed{(-3, -\frac{5}{8})}$$

VA: $x+1=0$
 $x=-1$

Domain: $x \neq -3, -1$

X-int: $x-2=0$
 $x=2$

$$\boxed{(2, 0)}$$

y-int: $y = \frac{0-2}{-4(0+1)} = \frac{-2}{-4} = \frac{1}{2}$

$$\boxed{(0, \frac{1}{2})}$$

HA: $y = \frac{1x^2}{-4x^2}$

$$\boxed{y = -\frac{1}{4}}$$

