5.3: Vertical Asymptotes, Holes, & Domain of Rationals

Steps:

1) Factor ☺
2) Replace
3) Cancel
4) look only at denominator!
   - ↳ factors that cancel make holes
   - ↳ factors that stay in denominator make vertical asymptotes
   - ↳ Domain ≠ holes, VA

• Ex 1) $f(x) = \frac{x^2-16}{2x^2-7x-4} = \frac{(x+4)(x-4)}{(2x+1)(x-4)}$

x^2-16

$\sqrt{x^2}=x \quad \sqrt{16}=4$

$(x+4)(x-4)$

$2x^2-7x(-4)$

$-8x^2$

$x + -8x = -7x$

$\frac{2x^2}{x} + \frac{x}{x} \mid \frac{-8x}{-4} - \frac{4}{-4}$

$x(2x+1) \mid -4(2x+1)$

$(2x+1)(x-4)$

Hole: $x-4=0$ $+4$ $+4$

Hole : $x=4$

VA: $2x+1=0$ -1 -1

$\frac{2x}{2} = \frac{-1}{2}$

VA: $x=-\frac{1}{2}$

Domain: $x \neq 4, -\frac{1}{2}$

• Ex 2) $f(x) = \dfrac{x-1}{(x-3)(x+1)}$

hole: none

VA: $x-3=0$ (+3 +3) $x+1=0$ (−1 −1)

VA: $x=3$, $x=-1$

Domain: $x \neq 3, -1$

• Ex 3) $f(x) = \dfrac{x(x-4)}{(x-4)}$

hole: $x-4=0$ (+4 +4)

hole: $x=4$

VA: none

Domain: $x \neq 4$