

## Guided Notes: 6.2 Equations of Circle

SWBAT graph circles on the coordinate plane and write the equations of circles in standard form.

### Standard Form of Circles

$$(x-h)^2 + (y-k)^2 = r^2$$

Center:

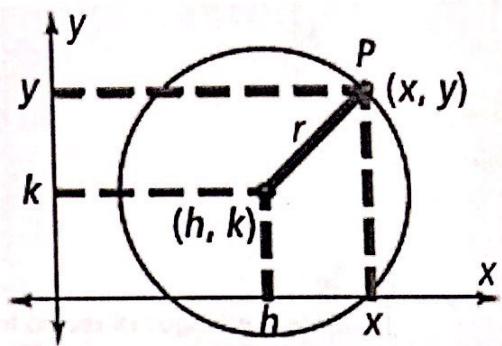
$$(h, k)$$

Radius:

$$r$$

Point on the circle:

$$(x, y)$$



**Example 1:** Write the equation of a circle with the given information.

a) Center  $(0,0)$ , Radius=10  
 $(x-0)^2 + (y-0)^2 = 10^2 \rightarrow x^2 + y^2 = 100$

b) Center  $(2, 3)$ , Diameter=12  
 $\frac{12}{2} = 6$   
 $(x-2)^2 + (y-3)^2 = 6^2$

**Example 2:** Determine the center and radius of a circle the given equation.

a)  $x^2 + y^2 = \frac{9}{4}$   
 center:  $(0, 0)$   
 $\sqrt{\frac{9}{4}} = r$   
 $r = \frac{3}{2}$

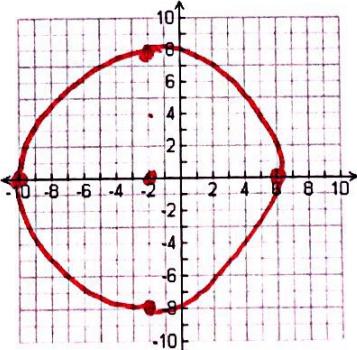
b)  $(x+3)^2 + (y-5)^2 = 81$   
 center:  $(-3, 5)$   
 $\sqrt{81} = r$   
 $r = 9$

c)  $(x+4)^2 + (y+6)^2 = 1$   
 center:  $(-4, -6)$   
 radius: 1

**Example 3:** Use the center and the radius to graph each circle.

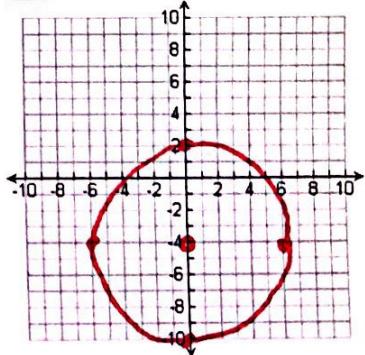
a)  $(x+2)^2 + y^2 = 64$

Center:  $(-2, 0)$   
 Radius:  
 $r^2 = 64$   
 $r = 8$



b)  $x^2 + (y+4)^2 = 36$

Center:  $(0, -4)$   
 Radius:  $r = 6$



## Writing an Equation with a Pass-Thru Point

**Step 1:** Substitute the center  $(h, k)$  into the equation

**Step 2:** Substitute the "pass through point  $(x, y)$ " into the equation for  $x$  and  $y$ .

**Step 3:** Simplify and solve for  $r^2$ .

**Step 4:** Substitute  $r^2$  back into the equation from Step 1.

**Example 4:** Write the equation of a circle with a given center  $(2, 5)$  that passes through the point  $(5, -1)$ .

$$(x-2)^2 + (y-5)^2 = r^2 \quad \text{and} \quad (5-2)^2 + (-1-5)^2 = r^2 \rightarrow r^2 = 45$$

$$(x-2)^2 + (y-5)^2 = 45$$

## Writing an Equation with Two Points on the Circle

### Midpoint Formula

Find the midpoint (radius) between the two endpoints, and then follow steps 1-4.

$$m = (x_1, y_1) = \left( \frac{x_1+x_2}{2}, \frac{y_1+y_2}{2} \right)$$

**Example 5:** Write the equation of a circle with endpoints of diameter at  $(-6, 5)$  and  $(4, -3)$ .

$$\left( \frac{-6+4}{2}, \frac{5-3}{2} \right) = (-1, 1) \quad (x+1)^2 + (y-1)^2 = r^2$$

$$(-6+1)^2 + (5-1)^2 = r^2$$

$$r^2 = 41 \quad (x+1)^2 + (y-1)^2 = 41$$

## Writing the Equation of a Circle in Standard Form

<b>Step 1:</b>	Group x's and group y's together.
<b>Step 2:</b>	Move any constants to the right side of the equation.
<b>Step 3:</b>	Use complete the square to make a perfect square trinomial for the x's and then again for the y's. *Remember, whatever you do to one side of the equation, you must do to the other!
<b>Step 4:</b>	Simplify factors into standard form of a circle!

**Example 5:** Write the equation of a circle in standard form. Then, state the center and the radius.

a)  $x^2 + y^2 + 4x - 8y + 16 = 0$

$$x^2 + 4x + \boxed{\phantom{0}} + y^2 - 8y + \boxed{\phantom{0}} = 0 + \boxed{\phantom{0}} + \boxed{\phantom{0}}$$

$$x^2 + 4x + \boxed{4} + y^2 - 8y + \boxed{16} = 0 + \boxed{4} + \boxed{16}$$

b)  $x^2 + y^2 + 6x - 4y = 0$

$$\frac{4}{2} = (2)^2 = 4 \quad -\frac{8}{2} = (-4)^2 = 16$$

$$(x+2)^2 + (y-4)^2 = 4$$

c)  $x^2 + y^2 - 6x - 2y + 4 = 0$

d)  $x^2 + y^2 + 8x - 10y - 4 = 0$

$$\text{.) } x^2 + y^2 + 6x - 4y = 0$$

$$x^2 + 6x + \boxed{9} + y^2 - 4y + \boxed{4} = \cancel{9} + \cancel{4} + \boxed{9} + \boxed{4}$$

$$\frac{6}{2} = (3)^2 = 9 \quad \frac{-4}{2} = (-2)^2 = 4$$

$$\boxed{(x+3)^2 + (y-2)^2 = 13}$$

$$\text{c) } (x-3)^2 + (y-1)^2 = 6$$