

6.4: Completing the square with circles

- Step 1: Group x's, group y's, & get #'s to the right.
2: take middle # (b), divide by 2, & square it.
Add that # to both sides of the equation.
3: build factors & combine numbers.

Ex 1) Find center & radius

$$x^2 + y^2 + 14x - 12y + 60 = 0$$

$$(x^2 + 14x + \frac{49}{4}) + (y^2 - 12y + \frac{36}{4}) = -60 + \frac{49}{4} + \frac{36}{4}$$

$$\frac{14}{2} = (7)^2 = 49$$

$$\frac{-12}{2} = (-6)^2 = 36$$

$$(x+7)^2 + (y-6)^2 = 25$$

center: (-7, 6)
radius: 5

Ex 2) $x^2 + y^2 - 4x - 8y - 55 = 0$

$$(x^2 - 4x + \frac{4}{4}) + (y^2 - 8y + \frac{16}{4}) = 55 + \frac{4}{4} + \frac{16}{4}$$

$$\frac{-4}{2} = (-2)^2 = 4$$

$$\frac{-8}{2} = (-4)^2 = 16$$

$$(x-2)^2 + (y-4)^2 = 75$$

center: (2, 4)

$$\begin{array}{r} 75 \\ \textcircled{5} \overline{) 15} \\ \underline{5} \\ \textcircled{5} \overline{) 30} \\ \underline{15} \\ \textcircled{5} \overline{) 15} \\ \underline{15} \\ 0 \end{array}$$

$\sqrt{3 \cdot 5 \cdot 5}$

$$r: 5\sqrt{3}$$

$$\text{Ex 3) } x^2 + y^2 + 6x - 9 = 0$$
$$(x^2 + 6x + \frac{9}{1}) + y^2 = 9 + \frac{9}{1}$$

$$\frac{6}{2} = (3)^2 = 9$$

$$(x+3)^2 + y^2 = 100$$

center: $(-3, 0)$
radius: 10