# 8.2: Properties of logs & solving using properties

- **Natural logarithm:** has a base of e ($\log_e y$), but instead we write $\ln y$.
- Properties (condensing & expanding)
  1) product rule: $\log_b M \cdot N = \log_b M + \log_b N$
  2) quotient rule: $\log_b \frac{M}{N} = \log_b M - \log_b N$
  3) Power rule: $\log_b M^{p} = p \cdot \log_b M$

Ex 1) Condense

a) $7 \ln x$

$\boxed{\ln x^7}$

b) $\log x + \log(x-2)$

$\log(x(x-2))$

$\boxed{\log(x^2 - 2x)}$

c) $\log 4x^3 - \log 2x$

$\log\left(\frac{4x^3}{2x}\right) \rightarrow \frac{\cancel{2} \cdot 2 \cdot \cancel{x} \cdot x \cdot x}{\cancel{2} \cdot \cancel{x}}$

$\boxed{\log(2x^2)}$

you try: $\log 2 + \log x - \log 3$

$\log 2x - \log 3$

$\boxed{\log \frac{2x}{3}}$

Ex 2) Expand

a) $\log 3 \cdot x^2$

$\log 3 + \log x^2$

$\boxed{\log 3 + 2\log x}$

b) $\log \frac{5}{x}$

$\log 5 - \log x$

you try: $\log \frac{(5x)^{54}}{3}$

$\log(5x)^{54} - \log 3$

$54\log(5x) - \log 3$

$\boxed{54\log 5 + 54\log x - \log 3}$

c) $\log \frac{9x^2}{2}$

$\log 9x^2 - \log 2$

$\log 9 + \log x^2 - \log 2$

$\boxed{\log 9 + 2\log x - \log 2}$

Ex 3) Solve

a) $\ln_e(3x-5) = 4$

$e^4 = 3x - 5$

$+5 \qquad +5$

$\frac{e^4+5}{3} = \frac{3x}{3}$

$\boxed{x = 19.87}$

b) $\log_6(4x+2) + \log_6 2 = 2$

$\log_6[(4x+2)(2)] = 2$

$\log_6(8x+4) = 2$

$6^2 = 8x+4$

$36 = 8x+4$

$32 = 8x$

$\boxed{x = 4}$

c) $\log_2(8x^3) - \log_2(2x) = 6$

$\log_2\left(\frac{8x^3}{2x}\right) = 6$

$\log_2(4x^2) = 6$

$2^6 = 4x^2$

$\frac{64}{4} = \frac{4x^2}{4}$

$\sqrt{16} = \sqrt{x^2}$

$\boxed{x = 4}$

$\frac{\cancel{8}^{4} x^{\cancel{3}\,2}}{\cancel{2}\cancel{x}}$

You try:

1) $\log_3(2x+1) + \log_3 3 = 4$

$x = 13$

2) $\log_2(x+6) - \log_2 x = 2$

$x = 2$

3) $\ln(4x-1) = 3$

$x = 5.27$