

## 8.3: Expand & Condense Logarithms, solve equations

Ex 1) Expand:

$$\begin{aligned} \text{a) } \log_5 3a^2b &= \log_5 3 + \log_5 a^2 + \log_5 b \\ &= \log_5 3 + 2\log_5 a + \log_5 b \end{aligned}$$

$$\begin{aligned} \text{b) } \log_8 \frac{x^4}{z^2} &= \log_8 x^4 - \log_8 z^2 \\ &= 4\log_8 x - 2\log_8 z \end{aligned}$$

$$\begin{aligned} \text{c) } \log \left( \frac{c^3 d}{g^4 h^7} \right) &= \log c^3 d - \log g^4 h^7 \\ &= \log c^3 + \log d - (\log g^4 + \log h^7) \\ &= 3\log c + \log d - 4\log g - 7\log h \end{aligned}$$

$$\begin{aligned} \text{d) } \ln(hk)^4 &= \ln h^4 k^4 \\ &= \ln h^4 + \ln k^4 \\ &= 4\ln h + 4\ln k \end{aligned}$$

$$\begin{aligned} \ln(hk)^4 &= 4\ln(hk) \\ &= 4(\ln h + \ln k) \\ &= 4\ln h + 4\ln k \end{aligned}$$

Ex 2) Condense:

$$\begin{aligned} \text{a) } 4\log_3 x + 9\log_3 y &= \log_3 x^4 + \log_3 y^9 \\ &= \log_3 x^4 y^9 \end{aligned}$$

$$\begin{aligned} \text{b) } \log 5 + \log 2 - \log x &= \log 5 \cdot 2 - \log x \\ &= \log 10 - \log x \\ &= \log \frac{10}{x} \end{aligned}$$

$$\begin{aligned}
 c) \ln x + 8 \ln y - \ln y - 3 \ln z &= \ln x + \ln y^8 - \ln y - \ln z^3 \\
 &= \ln x y^8 - (\ln y + \ln z^3) \\
 &= \ln x y^8 - (\ln y z^3) \\
 &= \ln \frac{x y^8}{y z^3} \\
 &= \boxed{\ln \left( \frac{x y^7}{z^3} \right)}
 \end{aligned}$$

Ex 3) Solve:

$$a) \log_4 (x+3) = 2$$

$$4^2 = x+3$$

$$16 = x + 3$$

$$x = 13$$

$$b) 6 = \ln 4m$$

$$\frac{e^6}{4} = \frac{4m}{4}$$

$$m = 100.857$$

$$c) \log_3 (2x+5) = \log_3 (7x-10)$$

$$2x+5 = 7x-10$$

$$15 = 5x$$

$$x = 3$$

$$d) 5^x = 18$$

$$\log_5 18 = x$$

$$x = 1.796$$

$$\begin{aligned}
 5^x &= 18 \\
 \ln 5^x &= \ln 18 \\
 x \ln 5 &= \frac{\ln 18}{\ln 5} \\
 x &= 1.796
 \end{aligned}$$