8.4: Solving Exponential Equations with Binomial Exponents

Type 1: Matching Bases

→ If the bases match, set the exponents =

Ex 1) a) $5^{2x} = 5^{x-3}$

$2x = x - 3$

$-x \quad -x$

$\boxed{x = -3}$

b) $3^{2x+1} = 27$

$3^{2x+1} = 3^3$

$2x + 1 = 3$

$-1 \quad -1$

$\frac{2x}{2} = \frac{2}{2}$

$\boxed{x = 1}$

c) $4^{5x} = 16^{x-1}$

$4^{5x} = 4^{2(x-1)}$

$5x = 2(x-1)$

$5x = 2x - 2$

$-2x \quad -2x$

$\frac{3x}{3} = \frac{-2}{3}$

$\boxed{x = \frac{-2}{3}}$

Type 2: non-matching bases

→ steps:

1) isolate the exponential expression
2) take log/ln of both sides
3) Power rule
4) solve for x

Ex 2) a) $e^{3x} = 50$

$\ln e^{3x} = \ln 50$

$3x \ln e^1 = \ln 50$

$\frac{3x}{3} = \frac{\ln 50}{3}$

$\boxed{x = 1.30}$

b) $\frac{5(10^{x+1})}{5} = \frac{12}{5}$

$10^{x+1} = 2.4$

$\ln 10^{x+1} = \ln 2.4$

$\frac{(x+1)\ln 10}{\ln 10} = \frac{\ln 2.4}{\ln 10}$

$x + 1 = 0.38....$

$-1 \quad -1$

$\boxed{x = -0.62}$

c) $2^{2x-1} + 4 = 25$

$-4 \quad -4$

$2^{2x-1} = 21$

$\ln 2^{2x-1} = \ln 21$

$\frac{(2x-1)\ln 2}{\ln 2} = \frac{\ln 21}{\ln 2}$

$2x - 1 = 4.39\ldots$

$+1 \quad +1$

$\frac{2x}{2} = \frac{5.39\ldots}{2}$

$\boxed{x = 2.70}$

You try: $\frac{2 \cdot 3^{x+2}}{2} = \frac{8}{2}$

$3^{x+2} = 4$

$\ln 3^{x+2} = \ln 4$

$\frac{(x+2)\ln 3}{\ln 3} = \frac{\ln 4}{\ln 3}$

$x + 2 = 1.26\ldots$

$-2 \quad -2$

$\boxed{x = -0.74}$

d) $5^{2x+1} = 6^{3x-4}$

$\ln 5^{2x+1} = \ln 6^{3x-4}$

$(2x+1)\ln 5 = (3x-4)\ln 6$

$2x\ln 5 + \ln 5 = 3x\ln 6 - 4\ln 6$

$-3x\ln 6 - \ln 5 \quad -3x\ln 6 \quad -\ln 5$

$2x\ln 5 - 3x\ln 6 = -4\ln 6 - \ln 5$

$x(2\ln 5 - 3\ln 6) = \dfrac{-4\ln 6 - \ln 5}{2\ln 5 - 3\ln 6}$

$\dfrac{2\ln 5 - 3\ln 6}{2\ln 5 - 3\ln 6}$

$x = 4.07$

*Graph $y' \; y^2$

$2^{nd} \rightarrow$ trace $\rightarrow 5$

$\rightarrow$ Enter $3x$