

GUIDED NOTES: Exponential Growth and Decay

EX1. Suppose 20 rabbits are taken to an island. The rabbit population then triples every year. The function $f(x) = 20(3)^x$ where x is the number of years, models this situation. How many rabbits would there be after 2 years?

$$y = a(b)^t$$

y : final amount

a : initial amount

b : growth/decay factor

t : time periods

$$f(x) = 20(3)^2$$

$$= \boxed{180 \text{ rabbits}}$$

EX2. Suppose two mice live in a barn. If the number of mice quadruples every 3 months, how many mice will be in the barn after 2 years?

time

b time period

$$2 \text{ years} = \frac{24 \text{ months}}{3 \text{ months}} = 8 \text{ three-month periods}$$

$$y = 2(4)^8$$

$$y = \boxed{131,072 \text{ mice}}$$

EX3. The value of an iPad decreases at 35% per year. If the starting price of the iPad is \$500, how much will the iPad be worth after 5 years?

t

n

$$1 - .35 = 0.65$$

\uparrow
 b

$$y = 500(0.65)^5$$

$$y = \boxed{\$58.01}$$

When can you buy the iPad for \$5?

$$\frac{5}{500} = \frac{500(.65)^t}{500}$$

$$.01 = .65^t$$

$$\log .65 \cdot .01 = t$$

$$t = \boxed{10.690 \text{ years}}$$

$$\ln 0.01 = \ln 0.65^t$$

$$\frac{\ln 0.01}{\ln 0.65} = \frac{t \ln 0.65}{\ln 0.65}$$

$$t = \boxed{10.690 \text{ years}}$$

EX4. Suppose the acreage of forest is decreasing by 2% per year because of development. After 6 years of development, there is 4,000,000 acres of forest remaining. How many acres were originally in the forest?

$$1 - .02 = 0.98$$

$$4,000,000 = a(0.98)^6$$

$$4,000,000 = a(.885\dots)$$

$$a = 4,515,475.988 \text{ acres}$$

EX5. Find a bank account balance to the nearest dollar, if the account starts with \$100, has an annual interest rate of 4%, and the money is left in the account for 12 years.

$$1 + 0.04 = 1.04$$

$$y = 100(1.04)^{12}$$

$$y = \$160.10 \rightarrow \$160$$

If you wanted to buy a new gaming system for \$250, when will you have enough?

$$250 = 100(1.04)^t$$

$$\frac{\ln 2.5}{\ln 1.04} = \frac{t \ln 1.04}{\ln 1.04}$$

$$t = 23.36 \text{ years}$$

Half-Life: Some unstable substances, like plutonium, decay over time. To measure the rate of decay, scientists refer to their "half life." The half life is the

time it takes for half to decay

EX6. The pesticide DDT was widely used in the United States until its ban in 1972. If the half-life of DDT is 15 years for 100 grams, how much DDT would be remaining after 45 years?

$$\frac{45 \text{ years}}{15 \text{ years}} = 3 \text{ years}$$

$$y = 100(.5)^3$$

$$y = 12.5 \text{ grams}$$